Rayleigh quotient maximization and its applications to social network analysis (50% seminar)

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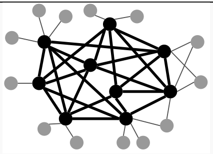
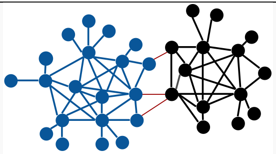
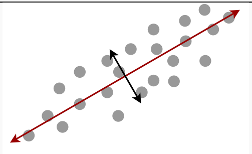
Introduction: Rayleigh quotient maximization over $\mathcal{T}$ (1/2)

Given a symmetric matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$ and a feasible set $\mathcal{T} \subseteq \mathbb{R}^n$, find any

$$\mathbf{u}^\star \in \operatorname{argmax} \left\{ \frac{\mathbf{x}^T \mathbf{A} \mathbf{x}}{\mathbf{x}^T \mathbf{x}} : \mathbf{x} \in \mathcal{T} \backslash \{\mathbf{0}\} \right\}.$$

Examples

densest subgraph [1]	2-community detection [9]	PCA [7]
$\mathcal{T} = \{0,1\}^n$	$\mathcal{T} = \{-1,1\}^n$	$\mathcal{T} = \mathbb{R}^n$
		

Introduction: Rayleigh quotient maximization over $\mathcal{T}$ (2/2)

Given a symmetric $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathcal{T} \subseteq \mathbb{R}^n$, find any

$$\mathbf{u}^\star \in \operatorname{argmax} \left\{ \frac{\mathbf{x}^T \mathbf{A} \mathbf{x}}{\mathbf{x}^T \mathbf{x}} : \mathbf{x} \in \mathcal{T} \backslash \{\mathbf{0}\} \right\}.$$

Discrete $\mathcal{T}$: graph applications

Questions of interests:

- What structures does $\mathbf{u}^\star$ capture?
- Is $\mathbf{u}^\star$ polynomial-time tractable?
- How well can we approximate $\mathbf{u}^\star$?

$\mathcal{T} = \mathbb{R}^n$: numerical linear algebra

Questions of interests:

- How to evaluate the quality of $\mathbf{u}^\star$?
- How well can we approximate $\mathbf{u}^\star$ under computational limitations?

Contributions

Given a symmetric $\mathbf{A} \in \mathbb{R}^{n\times n}$ and $\mathcal{T} \subseteq \mathbb{R}^n$, find any

$$\mathbf{u}^\star \in \operatorname{argmax}\left\{\frac{\mathbf{x}^T\mathbf{A}\mathbf{x}}{\mathbf{x}^T\mathbf{x}} : \mathbf{x} \in \mathcal{T}\backslash\{\mathbf{0}\}\right\}.$$

Part I - discrete $\mathcal{T}$

Studied a signed graph application.

- $\mathbf{u}^\star$ captures antagonistic patterns
- known to be APX-hard
- a provable approximation algorithm

Part II - $\mathcal{T} = \mathbb{R}^n$

Improved analysis of a numerical solver

- for the multiplicative gap $R(\hat{\mathbf{u}})$
- under memory-limited and pass-limited setting

(Part I) Tzeng et al. "Discovering conflicting groups in signed networks." In Proc. of NeurIPS 2020.
(Part II) Tzeng et al. "Improved analysis of randomized SVD for top-eigenvector approximation." In Proc. of AISTATS 2022.

Part I - an application in signed graphs

Given a symmetric $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathcal{T} \subseteq \mathbb{R}^n$, find any

$$\mathbf{u}^\star \in \operatorname{argmax} \left\{ \frac{\mathbf{x}^T \mathbf{A} \mathbf{x}}{\mathbf{x}^T \mathbf{x}} : \mathbf{x} \in \mathcal{T} \backslash \{\mathbf{0}\} \right\}.$$

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Conflicting group detection in signed graphs (1/8)

(Bonchi et al. 2019) 2-conflicting group detection

Given a signed adjacency matrix $\mathbf{A} \in \{-1, 0, 1\}^{n \times n}$, the 2-conflicting groups [3] are identified by the signs of

$$\mathbf{u}^\star \in \operatorname{argmax} \left\{ \frac{\mathbf{x}^T \mathbf{A} \mathbf{x}}{\mathbf{x}^T \mathbf{x}} : \mathbf{x} \in \{-1, 0, 1\}^n \backslash \{\mathbf{0}\} \right\} \quad (1)$$

$\mathbf{A}_{i,j}\mathbf{x}_i\mathbf{x}_j = 1$:

- $+$ intra-group edges
- $-$ inter-group edges

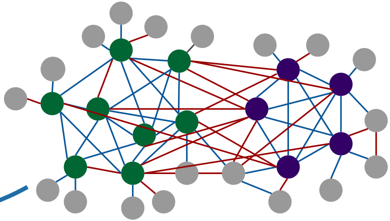
$\mathbf{A}_{i,j}\mathbf{x}_i\mathbf{x}_j = -1$:

- $-$ intra-group edges
- $+$ inter-group edges

antagonistic property:

intra-group: mostly $+$

inter-group: mostly $-$



Conflicting group detection in signed graphs (2/8)

(Bonchi et al. 2019) 2-conflicting group detection

Given a signed adjacency matrix $\mathbf{A} \in \{-1, 0, 1\}^{n \times n}$, the 2-conflicting groups [3] are identified by the signs of

$$\mathbf{u}^\star \in \operatorname{argmax} \left\{ \frac{\mathbf{x}^T \mathbf{A} \mathbf{x}}{\mathbf{x}^T \mathbf{x}} : \mathbf{x} \in \{-1, 0, 1\}^n \backslash \{\mathbf{0}\} \right\} \tag{1}$$

Hardness result of (1)

NP-hard: independently proven by

(Bonchi et al. 2019) reduction from Correlation Clustering

APX-hard proven by (Bhaskara et al. 2012)

version 1

reduction from Max k-AND

version 2

reduction from ratio version of Unique Games Conjecture

Conflicting group detection in signed graphs (3/8)

(Bonchi et al. 2019) 2-conflicting group detection

Given a signed adjacency matrix $\mathbf{A} \in \{-1, 0, 1\}^{n \times n}$, the 2-conflicting groups [3] are identified by the signs of

$$\mathbf{u}^\star \in \operatorname{argmax} \left\{ \frac{\mathbf{x}^T \mathbf{A} \mathbf{x}}{\mathbf{x}^T \mathbf{x}} : \mathbf{x} \in \{-1, 0, 1\}^n \setminus \{\mathbf{0}\} \right\} \tag{1}$$

Approximation algorithms

SDP-based (Bhaskara et al. 2012)

- general: $\tilde{\mathcal{O}}(n^{1/3})$
- bipartite: $\tilde{\mathcal{O}}(n^{1/4})$ (gap instance)

Eigenvector-based

- (Bonchi et al. 2019) $\mathcal{O}(n^{1/2})$
- (Tzeng et al. 2020) $\Omega(n^{1/2})$

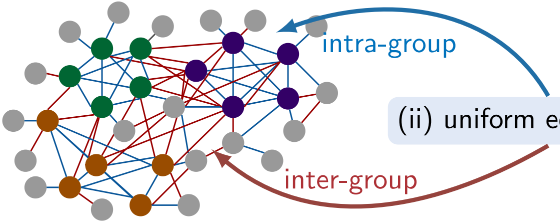
Conflicting group detection in signed graphs (4/8)

(Tzeng et al. 2020) formulation of k-conflicting group

k-conflicting groups are identified by the optimal solution to

$$\max_{\substack{\mathbf{X}\in\{0,1\}^{n\times k}\setminus\{\mathbf{0}\} \\ \mathbf{X}_{i,:}\in\{\mathbf{0},\mathbf{e}_1^T,\cdots,\mathbf{e}_k^T\},\forall i\in[n]}} \frac{\sum_{(i,j):\langle \mathbf{X}_{i,:},\mathbf{X}_{j,:}\rangle=1} \mathbf{A}_{i,j} + \frac{1}{k-1}\sum_{(i,j):\langle \mathbf{X}_{i,:},\mathbf{X}_{j,:}\rangle=0} \mathbf{A}_{i,j}}{\mathrm{Tr}(\mathbf{X}^T\mathbf{X})}.$$

(i) equally-sized groups



(ii) uniform edge density

Intuition: under (i)(ii), # inter-group edge $\approx (k-1)\times$ # intra-group edge

Conflicting group detection in signed graphs (5/8)

$$= \sum_{j=1}^{k-1} \frac{\|\mathbf{Y}_{:,j}\|_F^2}{\|\mathbf{Y}\|_F^2} \frac{\mathbf{Y}_{:,j}^T \mathbf{A}^{(j-1)} \mathbf{Y}_{:,j}}{\mathbf{Y}_{:,j}^T \mathbf{Y}_{:,j}}$$

(Tzeng et al. 2020) an equivalent objective of k-conflicting group

$$\max_{\mathbf{Y} \in \mathbb{R}^{n \times (k-1)} \setminus \{\mathbf{0}\}} \frac{\mathrm{Tr}(\mathbf{Y}^T \mathbf{A} \mathbf{Y})}{\mathrm{Tr}(\mathbf{Y}^T \mathbf{Y})} \text{ subject to } \mathbf{Y}_{i,j} = \begin{cases} c_j(k-j) & \text{if } i \in S_j \\ 0 & \text{if } i \in \cup_{h=1}^{j-1} S_h \text{ or } i \notin \cup_{h \in [k]} S_h \\ -c_j & \text{if } i \in \cup_{h=j+1}^{k} S_h \end{cases}, \tag{2}$$

where $\{c_j\}_{j \in [k-1]}$ are fixed constants, and $S_1, \cdots, S_k$ are any k disjoint groups.

(Tzeng et al. 2020) a sequential algorithm called SCG

Suppose $S_1, \cdots, S_{j-1}$ are found, we find $S_j = \{i \in [n] : \mathbf{u}_i^\star = k - j\}$ by solving

$$\mathbf{u}^\star \in \operatorname{argmax} \left\{ \frac{\mathbf{x}^T \mathbf{A}^{(j-1)} \mathbf{x}}{\mathbf{x}^T \mathbf{x}} : \mathbf{x} \in \{-1, 0, k-j\}^n \setminus \{\mathbf{0}\} \right\}, \tag{3}$$

where $\mathbf{A}^{(j-1)}$ is the adjacency matrix after removing $\cup_{h \in [j-1]} S_h$ and $\mathbf{A}^{(0)} = \mathbf{A}$.

Conflicting group detection in signed graphs (6/8)

(Tzeng et al. 2020) The subproblem in SCG

Let $q \in [k-1]$ and $\mathbf{A} \in \{0,1\}^{n \times n}$ be the (modified) adjacency matrix.

$$\mathbf{u}^\star \in \operatorname{argmax}\left\{\frac{\mathbf{x}^T\mathbf{A}\mathbf{x}}{\mathbf{x}^T\mathbf{x}} : \mathbf{x} \in \{-1, 0, q\}^n \setminus \{\mathbf{0}\}\right\}. \tag{3}$$

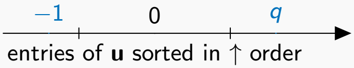
Solving (3): approximation algorithms

Eigenvector-based: let $\mathbf{u}$ be the leading eigenvector of $\mathbf{A}$.

- randomized: $\mathcal{O}(qn^{1/2})$-approx generalizes (Bonchi et al. 2019)

$$\tilde{\mathbf{u}}_i = \begin{cases} q \cdot \text{Bernoulli}(|\mathbf{u}_i|) & \text{if } \mathbf{u}_i > 0 \\ -1 \cdot \text{Bernoulli}(|\mathbf{u}_i|) & \text{if } \mathbf{u}_i < 0 \end{cases}$$

- deterministic:

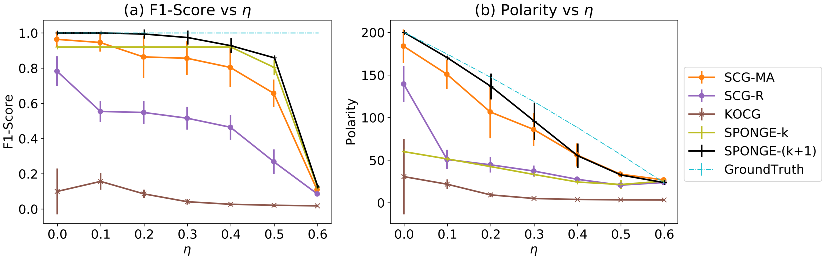


Conflicting group detection in signed graphs (7/8)

- Real-world networks:

	Bitcoin	WikiVote	Referendum	Slashdot	WikiConflict	Epinions	Wikipolitics
$\|V\|$	5 881	7 115	10 884	82 140	116 717	131 580	138 587
$\|E\|$	21 492	100 693	251 406	500 481	2 026 646	711 210	715 883
$\|E_-\|/\|E\|$	0.2	0.2	0.1	0.2	0.6	0.2	0.1
SCG-MA	**14.6**	**45.5**	**84.9**	**37.8**	**102.6**	**88.8**	**57.5**
SCG-R	5.0	9.7	39.8	7.3	16.2	39.4	5.5
KOCG [4]	4.4	5.5	8.8	2.6	4.5	8.7	4.8
SPONGE-k [5]	5.0	15.8	41.5	—	—	—	—
SPONGE-(k+1) [5]	0.8	1.0	1.0	—	—	—	—

- Synthetic:



Conflicting group detection in signed graphs (8/8)

Future work

- Can we improve the approximation guarantee to (3)?

$$\mathbf{u}^{\star} \in \operatorname{argmax}\left\{\frac{\mathbf{x}^T\mathbf{A}\mathbf{x}}{\mathbf{x}^T\mathbf{x}} : \mathbf{x} \in \{-1, 0, q\}^n \backslash \{\mathbf{0}\}\right\}. \tag{3}$$

- Can we design provable algorithm for k-conflicting group detection, (2)?
- What is the fundamental limit of the problem in synthetic model?
- Does our algorithm work well in sparse graphs?

Part II - an improved analysis for a numerical solver

Given a symmetric $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathcal{T} \subseteq \mathbb{R}^n$, find any

$$\mathbf{u}^\star \in \operatorname{argmax} \left\{ \frac{\mathbf{x}^T \mathbf{A} \mathbf{x}}{\mathbf{x}^T \mathbf{x}} : \mathbf{x} \in \mathcal{T} \backslash \{\mathbf{0}\} \right\}.$$

Part I - discrete $\mathcal{T}$

Studied a signed graph application.

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Improved analysis of Randomized SVD (1/8)

usage → the 2-conflicitng group algo by [3] becomes $\mathcal{O}(R(\hat{\mathbf{u}})^{-1}n^{1/2})$-approx

Our metric of interest: multiplicative gap $R(\cdot)$

Given a symmetric $\mathbf{A} \in \mathbb{R}^{n\times n}$ with the largest eigenpair $(\lambda_1, \mathbf{u}_1)$, $\lambda_1 > 0$, define

$$R(\hat{\mathbf{u}}) = \lambda_1^{-1}\frac{\hat{\mathbf{u}}^T\mathbf{A}\hat{\mathbf{u}}}{\hat{\mathbf{u}}^T\hat{\mathbf{u}}}$$

Prior: $\tilde{\mathcal{O}}(n)$-space numerical solvers

For any $\mathbf{A} \succcurlyeq 0$, w.h.p.

no guarantee | $R(\hat{\mathbf{u}}) = \Omega(1)$ → q

$o(\ln n)$-pass | $\Omega(\ln n)$-pass

state-of-the-art: for any $\mathbf{A} \succcurlyeq 0$,

Randomized SVD [6]: $R(\hat{\mathbf{u}}) \geq 1 - \mathcal{O}(\ln n/q)$

Block Krylov [8]: $R(\hat{\mathbf{u}}) \geq 1 - \mathcal{O}((\ln n/q)^2)$

[10] $R(\hat{\mathbf{u}}) = \Theta(1)$ impossible unless $q = \Omega(\ln n)$

Our analysis of Randomized SVD

For any $\mathbf{A} \succcurlyeq 0$, w.h.p.

$R(\hat{\mathbf{u}}) = \Omega\left(\left(\frac{\ln n}{n}\right)^{\frac{1}{2q+1}}\right)$ | $R(\hat{\mathbf{u}}) = \Omega(1)$ → q

$o(\ln n)$-pass | $\Omega(\ln n)$-pass

For some indefinite $\mathbf{A}$, w.h.p.

$R(\hat{\mathbf{u}}) = \Omega\left(\left(\frac{\ln n}{n}\right)^{\frac{1}{2q+1}}\right)$ | $R(\hat{\mathbf{u}}) = \Omega(1)$ → q

$o(\ln n)$-pass | $\Omega(\ln n)$-pass

Improved analysis of Randomized SVD (2/8)

$(q+1)$-pass $\mathcal{O}(nd)$-space

Intepreting Randomized SVD [6] for top-eigenvector approximation

Algorithm: RSVD $(\mathbf{A}, q, d)$

1. $\mathbf{Y} \leftarrow \mathbf{A}^q \mathbf{S}$ where $\mathbf{S} \sim \mathcal{N}(0,1)^{n \times d}$;
2. $\mathbf{Y} = \mathbf{QR}$;
3. $\mathbf{B} \leftarrow \mathbf{Q}^T \mathbf{AQ}$;
4. $\hat{\mathbf{u}} = \mathbf{Q}\, \mathbf{u}_1(\mathbf{B})$;
5. return $\hat{\mathbf{u}}$;

Fact: $\mathbb{E}[(\mathbf{u}_1^T \mathbf{S}_{:,j})^2] = \cdots = \mathbb{E}[(\mathbf{u}_n^T \mathbf{S}_{:,j})^2]$

(Step 1: line 1)

$\mathbf{Y}_{:,j} = \mathbf{A}^q \mathbf{S}_{:,j} = \sum_{i=1}^{n} \lambda_i^q (\mathbf{u}_i^T \mathbf{S}_{:,j}) \mathbf{u}_i, \ \forall j \in [d]$

(Step 2: line 2-4)

$\hat{\mathbf{u}} = \text{argmax}\{\mathbf{v}^T \mathbf{A} \mathbf{v} : \mathbf{v} \in \text{range}(\mathbf{Y}) \cap \mathbb{S}^{n-1}\}$

Effect of $q \uparrow$:

$\mathbf{Y}_{:,j}$ align more to eigenspace of λ_1

Effect of $d \uparrow$:

(i) $\uparrow$ the concentration around $\mathbb{E}[R(Y_{:,j})]$

(ii) $\uparrow \mathbb{E}[R(\hat{\mathbf{u}})]$ (we make this explicit)

Recall: $R(\hat{\mathbf{u}}) = \lambda_1^{-1} \frac{\hat{\mathbf{u}}^T \mathbf{A} \hat{\mathbf{u}}}{\hat{\mathbf{u}}^T \hat{\mathbf{u}}}$

Improved analysis of Randomized SVD (3/8)

$\geq e^{-\mathcal{O}(\frac{\ln n}{2q+1})} \geq 1 - \mathcal{O}(\frac{\ln n}{q})$

(Theorem 1) For $\mathbf{A} \succcurlyeq 0$, $R(\hat{\mathbf{u}}) = \left(\Omega\left(\frac{d}{n}\right)\right)^{\frac{1}{2q+1}}$ w.p. at least $1 - e^{-\Omega(d)}$.

Our technique: a reduction to random projection length

$$R(\hat{\mathbf{u}}) = \max_{\mathbf{a} \in \mathbb{S}^{d-1}} \frac{\sum_{i \in [n]} \alpha_i^{2q+1} \langle \mathbf{S}^T \mathbf{u}_i, \mathbf{a} \rangle^2}{\sum_{i \in [n]} \alpha_i^{2q} \langle \mathbf{S}^T \mathbf{u}_i, \mathbf{a} \rangle^2},$$

where $\alpha_i = \frac{\lambda_i}{\lambda_1}, \forall i \in [n]$.

$$\Rightarrow \cos^2 \theta(\mathbf{u}_1, \mathbf{S}) = \max_{\mathbf{a} \in \mathbb{S}^{d-1}} \frac{\langle \mathbf{S}^T \mathbf{u}_1, \mathbf{a} \rangle^2}{\sum_{i \in [n]} \langle \mathbf{S}^T \mathbf{u}_i, \mathbf{a} \rangle^2}$$

(Lemma [13]) For $\mathbf{v} \in \mathbb{S}^{n-1}$ and $d \ll n$, w.p. at least $1 - e^{-\Omega(d)}$,

$$\cos^2 \theta(\mathbf{v}, \mathbf{S}) = \Theta\left(\frac{d}{n}\right).$$

Improved analysis of Randomized SVD (4/8)

(Theorem 2) $\exists \mathbf{A} \succcurlyeq 0$ such that $R(\hat{\mathbf{u}}) = \mathcal{O}\left(\left(\frac{d}{n}\right)^{\frac{1}{2q+1}}\right)$ w.p. at least $1 - e^{-\Omega(d)}$.

(Theorem 3) For $\mathbf{A} \succcurlyeq 0$ with (i_0, γ)-power-law decay, $i_0 \in [n]$ and $\gamma > 1/2q$,

$$R(\hat{\mathbf{u}}) = \Omega\left(\left(\frac{d}{d+i_0}\right)^{\frac{1}{2q+1}}\right) \text{ w.p. at least } 1 - e^{-\Omega(d)}.$$

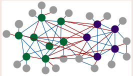
(Assumption 1) $\exists \kappa \in (0, 1]$ such that $\sum_{i=2}^{n} \lambda_i^{2q+1} \geq \kappa \sum_{i=2}^{n} |\lambda_i|^{2q+1}$.

(Theorem 4) For $\mathbf{A}$ with (i_0, γ)-power-law decay, $i_0 \in [n]$ and $\gamma > 1/2q$, and satisfying Assumption 1, there exists a constant $c_\kappa > 0$ such that

$$R(\hat{\mathbf{u}}) = \Omega\left(c_\kappa \left(\frac{d}{d+i_0}\right)^{\frac{1}{2q+1}}\right) \text{ w.p. at least } 1 - e^{-\Omega(\sqrt{d}\kappa^2)}.$$

Improved analysis of Randomized SVD (5/8)

e.g., $|S_1| + |S_2| = \Theta(n)$



Exploiting prior knowledge of large $\langle \mathbf{u}_1, \mathbf{1} \rangle^2$

Question: If $\langle \mathbf{u}_1, \mathbf{1} \rangle^2 = \Theta(n)$, is there a better choice of $\mathbf{S}$ other than $\mathcal{N}(0,1)^{n \times d}$?

Algorithm: RSVD($\mathbf{A}, q, d$)

$\mathbf{Y} \leftarrow \mathbf{A}^q \mathbf{S}$ where $\mathbf{S} \sim \mathcal{N}(0,1)^{n \times d}$;
$\mathbf{Y} = \mathbf{Q}\mathbf{R}$;
$\mathbf{B} \leftarrow \mathbf{Q}^T \mathbf{A} \mathbf{Q}$;
$\hat{\mathbf{u}} = \mathbf{Q}\, \mathbf{u}_1(\mathbf{B})$;
return $\hat{\mathbf{u}}$;

(Hint: $\mathbf{Y}_{:,j} = \mathbf{A}^q \mathbf{S}_{:,j} = \sum_{i=1}^{n} \lambda_i^q (\mathbf{u}_i^T \mathbf{S}_{:,j}) \mathbf{u}_i, \ \forall j \in [d]$)

Improved analysis of Randomized SVD (6/8)

Algorithm: RandSum($\mathbf{A}, q, d, p$)

1. $\mathbf{S}_1 \sim \mathcal{N}(0,1)^{n \times \lceil \frac{d}{2} \rceil}$, $\mathbf{S}_2 \sim \text{Bernoulli}(p)^{n \times \lfloor \frac{d}{2} \rfloor}$;
2. $\mathbf{S} \leftarrow [\mathbf{S}_1 \quad \mathbf{S}_2]$;
3. return RSVD($\mathbf{A}$,$\mathbf{S}$,q,d);

(Theorem 5) For $\mathbf{A} \succcurlyeq 0$, RandSum($\mathbf{A}$,$q$,$d$,$p$) returns $\hat{\mathbf{u}}$ satisfying

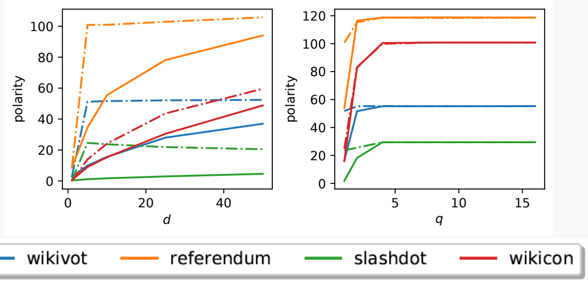
$$R(\hat{\mathbf{u}}) = \left(\Omega\left(\frac{\max\left\{d, \langle \mathbf{u}_1, \mathbf{1}_n \rangle^2\right\}}{n}\right)\right)^{\frac{1}{2q+1}} \quad \text{with prob.} \quad \geq 1 - e^{-\Omega(d)}.$$

Theorem 5 generalizes to indefinite $\mathbf{A}$ under an assumption similar to **Assumption 1**.

Improved analysis of Randomized SVD (7/8)

Experiment: 2-conflicting group detection [3, 11]

	WikiVot	Referendum	Slashdot	WikiCon
$\lvert V \rvert$	7 115	10 884	82 140	116 717
$\lvert E \rvert$	100 693	251 406	500 481	2 026 646
(γ, i_0)	(4.6, 15)	(4.5, 16)	(5.3, 17)	(2.8, 22)
κ	0.397	0.620	0.204	0.034
$\cos\theta(\mathbf{u}_1, \mathbf{1}_n)$	0.378	0.399	0.194	0.193



- RSVD: solid line
- RandSum: dashed line

Improved analysis of Randomized SVD (8/8)

Future work

- Do the results generalize to (row/column)-stochastic matrices?
- Do the results of RandSum hold for any non-centered subgaussian distributions?
- Can we extend the analysis to top-k eigenvectors approximations?
- What is the fundamental limit of $R(\hat{\mathbf{u}})$ for any q-pass $\tilde{\mathcal{O}}(n)$-space algorithm?
- Can we reduce the space complexity while keeping the same guarantees?

Summary

Part I - k-conflicting group detection

- We formulate the problem of k-conflicting group detection.
- We propose an algorithm that sequentially solves a sub-problem which generalizes the problem considered by Bonchi et al. [3] and Bhaskara et al. [2].
- We demonstrate the effectness of our algorithm.

Part II - improved analysis of Randomized SVD

- We improve the analysis of RSVD, in the regime of $o(\ln n)$ passes, and give the first analysis of $R(\cdot)$ for indefinite matrices.
- We study the property of Bernoulli random projection and demonstrate its usefulness to the task of conflicting group detection [3, 11].

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